SPECTRUM SHARING IN COGNITIVE RADIO

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>INTRODUCTION</strong></td>
</tr>
<tr>
<td>1.1</td>
<td>Motivation</td>
</tr>
<tr>
<td>1.2</td>
<td>The Cognitive Radio Technology</td>
</tr>
<tr>
<td>1.3</td>
<td>Organization of the Thesis</td>
</tr>
<tr>
<td>2</td>
<td><strong>Literature Review</strong></td>
</tr>
<tr>
<td>2.1</td>
<td>Spectrum Sharing in Single-Antenna Systems</td>
</tr>
<tr>
<td>2.2</td>
<td>Spectrum Sharing in MIMO Systems</td>
</tr>
<tr>
<td>3</td>
<td><strong>Adaptive Pricing for Efficient Channel and Power Allocation</strong></td>
</tr>
<tr>
<td>3.1</td>
<td>Problem Formulation</td>
</tr>
<tr>
<td>3.2</td>
<td>Channel and Power allocation Algorithm</td>
</tr>
<tr>
<td>3.3</td>
<td>Numerical Results</td>
</tr>
<tr>
<td>4</td>
<td><strong>Joint Channel and Power Allocation based on User Satisfaction</strong></td>
</tr>
<tr>
<td>4.1</td>
<td>Problem Formulation</td>
</tr>
<tr>
<td>4.2</td>
<td>Joint Channel and Power Allocation for Orthogonal Transmission</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Single Antenna</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Multiple Antennas</td>
</tr>
<tr>
<td>4.3</td>
<td>Power Allocation for Interference Transmission</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Single Antenna</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Multiple Antennas</td>
</tr>
<tr>
<td>4.4</td>
<td>Numerical Results</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Spectrum usage [2]</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Cognitive cycle [3]</td>
<td>7</td>
</tr>
<tr>
<td>2.1 MIMO channel model</td>
<td>17</td>
</tr>
<tr>
<td>3.1 Channel allocation</td>
<td>21</td>
</tr>
<tr>
<td>3.2 Sum data rates of IWF and PIWF. $K = 3$ and $N = 5.$</td>
<td>22</td>
</tr>
<tr>
<td>3.3 Sum data rates of IWF and OIWF. $K = 6,$ $N = 5$ and $P_{k}^{max} = 2W.$</td>
<td>32</td>
</tr>
<tr>
<td>3.4 Sum data rates of IWF and OIWF. $K = 6,$ $N = 5$ and $P_{k}^{max} = 1W.$</td>
<td>33</td>
</tr>
<tr>
<td>3.5 Sum data rates of IWF, OIWF and ES. $K = 2$ and $N = 3.$</td>
<td>34</td>
</tr>
<tr>
<td>3.6 Power allocations for IWF.</td>
<td>34</td>
</tr>
<tr>
<td>3.7 Power allocations for OIWF.</td>
<td>35</td>
</tr>
<tr>
<td>3.8 Power allocations for ES.</td>
<td>35</td>
</tr>
<tr>
<td>3.9 CR network for strong interference scenario.</td>
<td>37</td>
</tr>
<tr>
<td>3.10 Sum data rates of IWF and OIWF for strong interference. $K = 2$ and $N = 3.$</td>
<td>37</td>
</tr>
<tr>
<td>3.11 Power allocations for IWF in strong interference scenario.</td>
<td>38</td>
</tr>
<tr>
<td>3.12 Power allocations for OIWF in strong interference scenario.</td>
<td>38</td>
</tr>
<tr>
<td>3.13 CR network for weak interference scenario.</td>
<td>39</td>
</tr>
<tr>
<td>3.14 Sum data rates of IWF and OIWF for weak interference. $K = 2$ and $N = 3.$</td>
<td>39</td>
</tr>
<tr>
<td>3.15 Power allocations for IWF in weak interference scenario.</td>
<td>40</td>
</tr>
<tr>
<td>3.16 Power allocations for OIWF in weak interference scenario.</td>
<td>40</td>
</tr>
<tr>
<td>4.1 Data rate satisfaction</td>
<td>43</td>
</tr>
</tbody>
</table>
4.2 Power consumption satisfaction .................................................. 43
4.3 Total user satisfaction for orthogonal transmission with single antenna. 48
4.4 Total user satisfaction for interference transmission with single antenna. 52
4.5 CR network for weak interference scenario. ................................. 53
4.6 Comparison of total user satisfaction with weak interference. ......... 53
4.7 CR network for strong interference scenario. ............................... 54
4.8 Comparison of total user satisfaction with stronger interference. ....... 55
CHAPTER 1

INTRODUCTION

1.1 Motivation

Recent years have seen a rapid growth in the field of wireless communications mainly due to the advances in signal processing/semiconductor technologies and also due to the dramatic increase in user demands. Wireless communication relies heavily on a limited resource called radio spectrum, the access to which is regulated by the government agencies such as the Federal Communications Commission (FCC) of the United States and the U.K. office of communications (ofcom). These agencies allocate different frequency bands to different networks on a long term basis and this is known as the static spectrum allocation policy.

A recent survey by FCC has shown that the major drawback of the static allocation policy is the inefficient utilization of the available spectrum [1]. An analysis of the distribution of user activities shows that some frequency bands are heavily occupied, some frequency bands are partially occupied and some frequency bands are rarely used as shown in Fig. 1.1. With the increasing number of users day by day, there will come a time when there will be no available frequency bands for new users anymore, resulting in spectrum scarcity. Therefore, a more efficient spectrum assignment policy is required. In such a scenario, the concept of dynamic spectrum access has come into the picture.

A user, who owns a licensed piece of spectrum, is called a licensed user or primary user. A user, who does not own any licensed spectrum but still attempts to transmit data, is called a secondary user. Since the licensed users do not transmit all the
time, some spectra are sparsely used. The spectrum utilization can be improved if the secondary users could take advantage of these unused or rarely used licensed spectra. This concept of secondary users transmitting in licensed users’ spectrum without interfering with licensed users’ transmissions is called dynamic spectrum access. With the dynamic spectrum allocation policy, the frequency bands can be assigned to different networks dynamically in such a way that only when a user needs to transmit data, it is assigned a frequency band. Otherwise, the vacant frequency bands can be used by other users for transmission. Since the licensed spectrum may not be vacant all the time, the secondary users must continuously monitor the spectrum for any vacant band. When in usage of spectrum, the secondary user must vacate the spectrum immediately when the primary user returns. The current standard radios being used are not efficient enough to be deployed in the dynamic spectrum management scenario. Using the cognitive radio technology, the concept of dynamic spectrum access can be realized.
1.2 The Cognitive Radio Technology

In [3], cognitive radio is defined as “an intelligent wireless communications system which is aware of its environment and uses the methodology of understanding by building to learn from the environment and adapt to statistical variations in the input stimuli”. There are two primary objectives: “to provide highly reliable communication whenever and wherever needed and to improve radio spectrum utilization”. In simple terms, a cognitive radio scans the radio environment for spectrum holes (vacant frequency bands). After detecting the spectrum holes, it selects the best available spectrum hole based on the requirements for transmission and vacates the channel when the licensed user returns.

A cognitive radio is built on software defined radio. A software defined radio is a wireless communication system that implements communication functions such as modulation, demodulation, amplifiers, etc. in software. Additionally, a cognitive radio can sense its environment, track changes and possibly react upon its findings. The major tasks of cognitive radio can be classified into three categories:

1) Radio scene analysis: In this task, we deal with detection of spectrum holes and also determination of the interference temperature. The interference temperature provides a measure of the acceptable amount of interference at a particular frequency band at the receiver side, so any transmission in the band is considered to be satisfactory if the noise is below the interference limit and is considered to be harmful if the noise is above the interference temperature limit [3]. The techniques used for detection of spectrum holes can be classified into transmitter detection, cooperative detection and interference based detection methods [2]. The research issues or the challenges we face in this task include:

- Detection of spectrum holes in a short duration of time. With the fast changing environment, it is very important that the spectrum holes are detected quickly.
The primary users may switch on and off randomly and possibly frequently. In the mean time, this vacant spectrum can only be used if it is detected by the secondary user.

- Ability to predict a vacant spectrum in near future. For a faster dynamic spectrum access, it is very important to predict when a spectrum might be vacant in future. This can be achieved by continuous monitoring of the spectrum but this involves a lot of energy cost. When one does not have any data to transmit currently, the question arises if it is worthwhile to monitor the spectrum continuously by investing a lot, expecting that one might have some data to transmit in future?

2) Channel state estimation: In this task, the focus is on determining the channel capacity for which the state of the channel also needs to be determined.

3) Spectrum management: The main goal of this task is efficient spectrum sharing of the vacant channels detected in the radio scene analysis stage. This can be achieved by an appropriate channel allocation scheme, power allocation scheme and proper selection of modulation strategies, etc. The research issues or the challenges we face in this task include

- Transmission power levels determine the achieved data rates, i.e., the more the transmission power, the higher is the achieved data rate. However, increasing the transmission power of one user, say user A will cause more interference to another user, say user B. To negate this interference, user B will increase its transmission power which in turn will cause more interference to user A. This may end up with each user transmitting with more power, which is a waste of energy. The main challenge here is to find the optimal transmission power levels of all users. The power control algorithms have been studied with different objective functions such as maximization of the sum data rate [4] and the sum
utility function \cite{5}. The challenge here is to design the appropriate algorithm based on user’s requirements.

- Hand-off mechanism: Suppose a licensed user is not using its allocated frequency band, then this vacant frequency band can be assigned to a cognitive radio (CR) user. When the CR user is still transmitting its data and the licensed user returns for its frequency band access, the CR user must vacate the band. In such a scenario, the CR user has to switch to another frequency band immediately to avoid any delays in its transmission and resume its transmission. This is known as the hand-off mechanism \cite{2}. Spectrum hand-off can also occur when the CR user moves from one location to another, and if in this new location the previously assigned frequency band is not available, the CR user must switch to another frequency band.

- There is generally a maximum allowable transmission power on each band for secondary users. This constraint is for protecting the primary users. That is the allocated power to each CR user is set in such a way that the total collective interference due to all CR users at primary user’s receiver must not cross an acceptable limit. In such a case, the transmission power of each CR user depends on the total number of CR users transmitting in the same channel. Since in a practical scenario, the composition of CR users’ changes from time to time, setting a neighborhood dependent transmission is very difficult.

- Channel allocation scheme: There are two possible ways of allocating a channel. One is that, only one CR user can transmit on a particular channel at a time, i.e., orthogonal transmission. The other possible way is that, multiple CR users can simultaneously transmit on a channel, i.e., interference transmission. The interference transmission mode can be an advantageous over orthogonal transmission because in interference transmission each CR user can have more
channels for transmission. As the data rate increases with the increase in bandwidth, each CR user needs less power to attain the same data rate. But the drawback of interference transmission is that if we have many users, and the interference may increase in each channel resulting in an inefficient usage of channels. In such a case, orthogonal transmission may be more suitable. The challenge here is to design such an algorithm that chooses between orthogonal and interference transmission mode in a dynamic environment.

- In the cognitive radio technology, a common control channel is generally needed to provide functionalities such as setting up a communication link between the transmitter and the receiver, to communicate with a central unit or to exchange sensing information with other CR users. The CR users have to vacate a channel if the primary user comes back for transmission. This applies even to a control channel. Hence, a particular channel cannot be fixed as a control channel forever and the control channel can vary from time to time. Therefore the implementation of a common control channel for all the CR users is a challenging task.

The interaction of the three main tasks with one another and with the environment is pictured in the cognitive cycle in Fig. 1.2. The cognitive cycle employs a state diagram to alter its actions in response to the changes in the environment. The states are the three tasks, and the transition from one state to another state depends on the input parameters. These input parameters can be output of other tasks or from the environment. Task 1 and task 2 are carried out at the receiver, whereas task 3 is carried out at the transmitter. For task 3 to be implemented at the transmitter side, we need the data from the receiver side, and this is achieved by having a feedback channel between the receiver and transmitter.
1.3 Organization of the Thesis

In this thesis, the main focus is on the problem of efficient spectrum sharing. The vacant frequency bands detected by the CR user in the radio scene analysis stage are spread over different frequency ranges. The characteristics of these frequency bands vary from time to time according to the changing environment. Spectrum sharing in CR is not only just assigning vacant frequency bands to each CR user but also a careful selection of frequency bands which is needed for efficient spectrum utilization. For this selection of frequency bands to be possible, the analysis of the spectrum is necessary. Each frequency band is analyzed for the information about the users that are transmitting in this band, the amount of interference the CR user experiences and the amount of power the CR user can transmit, while the interference they cause to primary user is below an acceptable limit.

The remainder of the thesis is organized into four chapters. Chapter 2 discusses
and categorizes the related work done by different researchers in spectrum sharing. In Chapter 3, we extend the idea in [6] and derive an adaptive pricing vector in a setting with multiple MIMO interference channels. With the objective of maximizing the sum data rate, we solve the problem of $K$ pairs of CRs sharing $N$ vacant channels by designing a distributed iterative algorithm that performs the channel and power allocation.

Chapter 4 introduces a new metric denoting user satisfaction. With the objective of maximizing the total user satisfaction, joint channel and power allocation algorithms are designed for both single-antenna and multiple-antenna configurations for both ways of transmission, orthogonal and interference. Experiments are conducted to demonstrate the efficiency of the proposed algorithms and the performance gains through allowing interference transmission and/or using multiple antennas. In the last chapter, we conclude the thesis and discuss the practical implementation of the proposed algorithms as part of the future work.
CHAPTER 2

Literature Review

This chapter describes the work done by different researchers on the spectrum sharing problem. There are generally two ways of allocating a channel: one is orthogonal transmission, where only one CR user can transmit on a channel at a time, and the other is interference transmission, in which multiple CR users can transmit on a channel simultaneously. The first section discusses the work done on systems with single antenna at transceivers, and Section 2.2 explains the basic concepts of a multiple-input multiple-output system (MIMO) and the work done in the MIMO system.

2.1 Spectrum Sharing in Single-Antenna Systems

In the past two decades, many algorithms have been developed to tackle the spectrum sharing problem resulting in various technologies. Based on the architecture, these algorithms can be classified into two categories, namely the centralized processing and the distributed processing [7]. In centralized processing, a central unit decides the spectrum and power assignment policies for all the users in the network. All the terminals in the network forward their data like spectrum, power and interference information to the central unit and the central unit implements the spectrum sharing algorithm using all the information. The central unit then broadcasts the channel indices that each terminal can use as well as power that can be allocated through a control channel. In decentralized processing, the terminals implement a distributed algorithm for the spectrum sharing. The algorithm can be implemented based on the locally observed information by the terminal and/or information gath-
ered through exchange with other users in the neighborhood. With the advantages of lower bandwidth requirements and faster adaptability to changes in the environment over centralized processing, the decentralized processing has gained more attention and many algorithms are being developed based on it.

The algorithms based on centralized and distributed architectures for spectrum sharing can be designed using different approaches such as Game Theory [5], genetic algorithms [8], biologically-inspired algorithms [9], graph coloring approach [10], etc. Game theory is a powerful tool in the field of economics. It has been used to analyze situations where users compete with each other to access the same resource. There are many models of the game such as potential game, cooperative game, symmetric game, non-cooperative game, etc. In the case of a radio network, the whole network can be modeled as a cooperative game or a non-cooperative game. A general form of a game, which can be applied to both cooperative and non-cooperative games can be expressed as

$$G = [K, P, \{U_k(\cdot)\}]$$

It has three components, where $G$ denotes a particular game, $K$ is the number of players (users in a radio environment), $P$ is the strategy set (the power vector representing allocated powers to users in a radio environment) and $U_k(\cdot)$ denotes the utility function.

In the field of economics, the utility function is defined as “a measure of the satisfaction experienced by a person using a resource” [11]. In the case of a radio network, the quality of service received from the usage of the network resources is expressed quantitatively in the form of a utility function.

1) Cooperative game. In this type of a game, players cooperate with each other to attain a better solution. The players negotiate and bargain with each other. Many solutions [12] are available to the cooperative game theory, among them the Nash Bargaining Solution (NBS) provides an efficient and fair spectrum allocation [13][14].
The solution proposed by Nash in [15] is based on axioms and the resultant bargaining solution satisfies these axioms.

Cooperative games have been applied to different problem formulations with the importance given to the fairness among users. [16] formulates a cooperative game to analyze the spectrum sharing problem in a multi-hop wireless network, with each user having a limited power. The users are segregated into overlapping groups and users in the same group cooperate and exchange information. A distributed algorithm for interference transmission is designed with two goals: efficiency and fairness among users. The algorithm achieves a power allocation solution satisfying the NBS axioms.

In [17], the resource allocation problem to multiple OFDMA users is studied. Similar to [16], a distributed algorithm with a goal of achieving fairness is proposed but for an orthogonal transmission and also an additional constraint of a minimum data rate requirement to each user is considered. The algorithm consists of two stages. In the first stage, by employing the Hungarian method the users are aggregated into groups of size two and in the second stage, a two-user algorithm based on the water-filling solution is employed for subcarrier and power allocation.

2) Non-Cooperative game. In this form of a game, each user maximizes its own utility function and behaves selfishly, neglecting the interference it causes to other users. As the name itself suggests, there is no cooperation among the users. Nash equilibrium (NE) is the most widely used solution of a non-cooperative game [18], “wherein no player would rationally choose to deviate from their chosen actions because the utility under that condition is larger than what they could achieve by deviating” [19].

Due to the noncooperative nature of the game, the resultant Nash equilibrium (NE) solution, if exists, may not be Pareto optimal [20][21]. In fact, the solution that achieves the maximum sum data rate is essentially a Pareto optimal solution. To improve the NE solution, the concept of pricing is introduced to prevent the users from behaving selfishly. In economics, there are different types of pricing policies such
as usage-based pricing, access-based pricing and priority-based pricing. In the case of a radio network, usage-based pricing is used, i.e., the amount one pays is proportional to the amount of resource one uses, and here the resource being used is power. The optimal pricing functions are generally difficult to obtain and heuristic pricing factors are often adopted for simplicity. The net utility function, $\hat{U}(\cdot)$ with a pricing function is expressed as

$$\hat{U}(P_k) = U_k(\cdot) - C$$

where $C$ denotes the pricing function.

In recent years, researchers have formulated different types of utility functions to solve the decentralized architecture of spectrum sharing. In [22], a distributed iterative water-filling algorithm is designed in a digital subscriber line system. The outcome of the algorithm is a channel and power allocation solution in such a way that each user attains its target data rate. The utility function is the data rate itself and no pricing function is considered, resulting in an inefficient NE solution. In [23], the authors solved the uplink power control problem in a code-division multiple-access wireless data system. A utility function as a function of the bit error rate is used and each user maximizes its own utility. An asynchronous algorithm is designed for power allocation. As the solution obtained is an inefficient NE solution, the authors used a linear pricing function to improve the solution. Although the resultant solution is improved with the usage of pricing function, it is not Pareto optimal. [24] studies the power allocation problem in cellular systems. The utility is a function of the signal to interference ratio. Similar to [23], a linear pricing function is employed to improve the NE solution. Compared to the aforementioned papers, [25] presents the problem of distributing the total available power across the multi-hops in an interference relay channel instead of allocating the power to different channels. An asynchronous distributed algorithm is designed to achieve a NE solution and is proved to be Pareto-optimal in only a few cases. However, these solutions are generally not
Pareto-optimal in terms of the sum data rate because the algorithm maximizes each user’s data rate and neglects the interference to other users.

In [26], the authors design an asynchronous distributed algorithm, which allocates the total available power of each user on different channels in a wireless network. Maximization of the sum utility of all users in the wireless network is considered and each user’s utility is an exponential function of the data rate. An interference pricing factor for each channel is derived and is implemented in the algorithm with little information exchange among users. The disadvantage in [22]-[25] is overcome by employing this interference pricing factor, as this new form of pricing function results in a unique optimal solution. The interference pricing factor signifies that each user pays a price for causing interference to other users on the same channel.

More recently, research has been conducted on spectrum sharing in the CR technology. Compared to spectrum sharing in other technologies, the noticeable factors in the CR technology are the presence of PUs and how the CR users take measures to protect the PUs. Generally, there is an interference limit that primary users (PU) can tolerate or the CR user may employ some pricing or coding techniques to mitigate the interference to PUs. At the same time, the CR users must maximize their own efficiency. Based on these constraints, the spectrum sharing algorithms are designed in the CR technology. Along with the classifications mentioned above, spectrum sharing in CR can be further divided into three communication models [27], based on how the CR users access the spectrum: underlay, overlay and interweave.

1) In the underlay model, the CR users can transmit simultaneously with the PUs provided that the interference they cause at the PU’s receiver is below a certain acceptable threshold. [28] describes the problem of subcarrier, bit and power allocation to a CR user employing an OFDM modulation technique. The resource allocation is formulated as a multidimensional knapsack problem under constraints of limited available power to the CR user and the tolerable interference of the PU. The authors
design a max-min algorithm to solve this problem in a network consisting of four PUs and one CR and achieve close to optimal solution.

In [28], the scenario of multiple CR users coexisting in a network is not discussed. In [6], coexistence of multiple CR users is considered. The authors introduce two distributed algorithms for interference transmission in a non-cooperative game under two constraints, limited available power to each user and tolerable interference of the PU. The efficiency of the NE solution is improved by using pricing techniques. Similar to [26], a user dependent pricing factor is employed. The pricing factor is derived by comparing individual user optimization problem with social optimization problem with the help of the Lagrange multipliers technique. The pricing factor for each user is calculated using the information exchanged with other users. The Pareto optimal solution is obtained by the price based iterative water-filling algorithm (PIWF). There are two versions of PIWF, one is the sequential price based iterative water-filling algorithm (SPIWF) and other is the parallel price based iterative water-filling algorithm (PPIWF). In the SPIWF algorithm, the power allocations of the CR users are evaluated in a sequential order and each CR user updates its power allocation sequentially, while in PPIWF, the CR users update their power allocation simultaneously. As [28] and [6] follow the underlay model, interference from PUs is considered during the evaluation of the algorithm.

In [4], the author presents an algorithm for joint channel and power allocation. Different from [6], this paper is based on the centralized architecture and for orthogonal transmission. There is an access point controlling the transmissions of CR users and this access point also has the information about the vacant channels, power gains of the channels for different users and acceptable amount of interference at the receiver of PU due to CR users. The author designs an explicit channel allocation algorithm, which is implemented at the access point. Under the constraints of limited available power and tolerable interference to PU, each CR user allocates power to its
assigned channels by employing the water-filling solution.

2) In the overlay model, the CR users can transmit simultaneously with the PUs but in this model the CR users either assist the PU transmissions or make use of the PU messages for its benefit. [29] studies the achievable rate region when the simultaneous transmission of CR user and PU is allowed. The CR user has the knowledge of the PU’s message and uses this information to improve its own data rate. CR user employs the dirty paper coding technique to mitigate the interference at its receiver. Similar to [29], [30] discusses the achievable rate of a CR user, when both PU and CR user coexist in a channel. But the CR user uses a portion of its power to relay the PU’s signal to PU’s receiver and the remaining power to transmit its own signal. The CR user employs the dirty paper coding technique to reduce the interference at its receiver, caused by the PU’s transmitter.

3) In the interweave model, the CR users access the spectrum opportunistically when the PU is not transmitting. Recent studies have shown that the assigned spectrum to PU is vacant most of the time. The CR users take advantage of this temporarily vacant spectrum to communicate with other CR users.

In [3], Haykin extends the idea in [22] to a cognitive radio network and designs a distributed iterative water-filling algorithm for channel and power allocation. With no pricing technique employed, the solution is an inefficient NE solution. In [31], the authors make two modifications to the utility function introduced by Goodman in [11]. The utility is a function of the efficiency function and the efficiency function varies with the bit error rate. As there are different bit error rate expressions for different modulation techniques, we have different efficiency functions. The first modification is that, a common sigmoid efficiency function is employed for all modulation techniques. The advantage of this common sigmoid efficiency function is that, irrespective of the type of modulation, all the users have a common efficiency function and it depends only on the users’ signal to interference ratio. The second modifica-
tion is that, the pricing function is a function of the channel gain. The new pricing function essentially means that each user pays a price proportional to the amount of power it transmits with and also the channel gain from the user to the base station. Each user maximizes this new utility and finds the power allocation solution. There is an obvious improvement in the result but still the solution is not Pareto-optimal.

2.2 Spectrum Sharing in MIMO Systems

With the ever increasing demand for higher data rates and with the limitations of the conventional wireless system due to fading and interference from other users transmitting in the same frequency band, the multiple-input multiple-output (MIMO) technology has gained great attention. The advantage of a MIMO technology is that by exploiting the inherent diversity of using multiple antennas at transceivers, the capacity increases almost linearly with the number of antennas [32]. This section focuses on the basic concepts of a MIMO system, such as the system model, the general capacity expression and also discusses the work done in MIMO systems.

System Model

Consider a MIMO channel with two users each having $t$ antennas at the transmitter and $r$ antennas at the receiver. The input-output relation of user 1 in a MIMO system is given by

$$y_1 = H_{11}s_1 + H_{21}s_2 + n$$  \hspace{1cm} (2.1)

where $y_1$ is the $r \times 1$ received signal vector of user 1, $s_1$ is the $t \times 1$ transmitted signal of user 1, $H_{11}$ is the $r \times t$ channel matrix from the transmitter to the receiver of user 1, $H_{21}$ is the $r \times t$ channel matrix from the transmitter of user 2 to the receiver of user 1 and $n$ is the noise. The second part in (2.1) is the interference from user 2. Similarly the input-output relation of user 2 can be expressed. The MIMO system with two users is pictured in Fig. 2.1.
In a MIMO system, the capacity [32] of user 1 is given by
\[
C = \log_2 |I + H_{11}P_1H_{11}^*Q_1^{-1}|
\]
I is an identity matrix of a proper size. (·)' stands for transpose conjugate. \(P_1\) is the transmission power matrix of user 1. \(Q_1\) is noise plus interference from user 2 to user 1,
\[
Q_1 = \sigma^2 I + H_{21}P_2H_{21}^*
\]
\(\sigma^2\) is the power of channel noise. The above system model of two users in a single channel can be extended to multiple users transmitting in multiple channels.

[33] presents the power control and antenna selection problem for a single user in varying channel conditions. To adapt with the varying channel, Kalman filtering is used at the receiver to estimate the channel and then this information is sent to the transmitter via feedback. In [34]-[36], a channel sharing among multiple users is studied, where channel conditions are assumed to be constant during the evaluation of the algorithm. In [34], power allocation is derived using the iterative water-filling (IWF) algorithm, which leads to an NE solution. The MIMO capacity is investigated un-
der different interference environments and different assumptions of the channel state information. [35] proposes a power allocation algorithm to solve the interference management issue in a wireless ad hoc network. The algorithm is based on the Lagrange multipliers technique in a non-cooperative game formulation. By employing a fixed pricing factor, a link shutdown mechanism is developed to reduce interference and the NE solution is improved compared to [34]. [36] presents a subcarrier and power allocation problem for a downlink MIMO-OFDM system. The users employ both OFDM and MIMO techniques. The block diagonalization technique is employed along with the iterative water-filling solution to improve the solution. Block diagonalization is a precoding technique used to mitigate interference to other users.

In [37]-[39], the channel is shared between the PU and CR users, i.e., an underlay model. In [37], an interference alignment technique is proposed to mitigate the interference to PU. The idea behind the interference alignment technique is that, under power constraints the IWF algorithm results in some unused subchannels, i.e., there is no transmission by the PU in the corresponding spatial directions. Based on this idea, the precoder matrix of the CR user is generated and this will lead the CR user to transmit in the unused spatial directions. In [38], the nonconvex problem of maximizing the sum capacity of an interference channel is converted into a convex problem by using duality techniques. An algorithm based on the iterative water-filling solution is designed to allocate power in order to maximize the sum capacity. [39] considers a non-cooperative game in which PU imposes null and soft constraints on CR users. The null constraints indicate that no CR transmission is allowed in few subspaces and the soft constraint indicates an tolerable interference in some subspaces. A distributed IWF algorithm is designed based on these constraints resulting in an efficient NE solution.

[40] considers the problem of power allocation to CR users equipped with both single and multiple antennas in an overlay system. The authors prove that only a limited
gain is achieved by using a simple dirty paper coding (DPC) technique, because in DPC, time is wasted on the CR user decoding and forwarding the PU’s transmitted signal to the receiver of PU. This gain is improved by using the zero forcing approach, which mitigates the interference to PU instead of relaying the PU’s message. However, the above solutions may not be Pareto optimal. The dirty paper coding technique may be a theoretically optimal solution in mitigating interference to other users and achieving an improved capacity. But it is highly complicated and difficult to implement practically. Zero forcing and block diagonalization are less complicated techniques compared to dirty paper coding, but result in sub-optimal solutions. In the following chapter, an efficient spectrum sharing algorithm for a MIMO system is designed based on pricing techniques. The algorithm is less complicated, needs little information exchange between the users and achieves a Pareto optimal solution if interference is treated as noise.
CHAPTER 3

Adaptive Pricing for Efficient Channel and Power Allocation

In this chapter, we consider the problem of spectrum sharing among multiple cognitive radio (CR) users in an underlay system. Each user consists of a pair of transmitter and receiver and assuming each transmitter has data to send to its receiver, will be looking for empty frequency bands to transmit. These multiple CR users coexist in a network and compete for the same spectrum. The CR users can transmit simultaneously in all the channels provided that the interference they cause to primary users is below a certain acceptable limit. Each user has a limited amount of power, which it can distribute among different channels. The spectrum sharing problem is to allocate the vacant channels to the CR users such that the objective function, be it total data rate or total user satisfaction is maximized under a set of power constraints. A possible channel allocation for a network consisting of 3 users and 5 channels is depicted in Fig. 3.1. The dark box at (1,3) in the figure indicates that the channel centered at $f_1$ is occupied by user 1 ($U_1$) and white box at (1,2) indicates that user 2 does not transmit on channel 1. Similarly other boxes can be interpreted.

As discussed in Chapter 2, [6] solves the problem of channel and power allocation for a single antenna system. A user dependent pricing factor is derived, such that each user employs a different pricing factor on each channel proportional to the interference it causes to other users. The derived formula for the pricing factor of user $k$ on channel $n$ is given as

$$\lambda_{kn} = \sum_{j \neq k, j=1}^{K} \frac{h_{jjn} \cdot P_{jn} \cdot h_{kjn}}{M_{jn}(M_{jn} + h_{jjn}P_{jn})}$$  \tag{3.1}$$

where $h_{jjn}$ is the channel gain of user $j$, $P_{jn}$ is the power allocated to user $j$ and $M_{jn}$
Figure 3.1: Channel allocation

is the interference of user $j$ on channel $n$. Based on the pricing factor, a price-based iterative water-filling (PIWF) algorithm is designed to achieve the Pareto optimal solution. The sum data rates achieved by both classical IWF and PIWF algorithms are displayed in Fig. 3.2. From the figure, we observe that the achieved sum data rate by PIWF is much larger than that of IWF. In IWF, each user allocates the total available power on all channels, causing high interference to other users. This negative effect reduces the system efficiency and eventually itself. Whereas in PIWF, a variable power level is adopted due to the pricing factor, signifying the punishment to each user proportional to the interference it causes to other users, thus reducing the interference to other users and resulting in a higher sum data rate.

In this chapter, we consider the problem of channel and power allocation to multiple CR users but the CR users are equipped with multiple antennas at both transmitters and receivers. Similar to [6], a non-cooperative game with pricing is formulated to tackle the problem. We show that, to achieve the maximum sum data rate, each user should incorporate a vector of adaptive pricing factors on each channel based on the transmission power of other users. The formula of the optimal pricing vector
is derived and a distributed iterative algorithm is designed through small amount of information exchange between users. The simulation results demonstrate the performance gain of the proposed algorithm over the classical IWF algorithm without pricing and intuitive pricing formulations.

3.1 Problem Formulation

We assume the availability of a set of $N$ channels as a result of periodic spectrum sensing [41]-[43]. $K$ pairs of SUs with cognitive radio capabilities are randomly deployed in a unit area and are trying to access the vacant channels. Each user has a constraint on the maximum total transmission power, $P_{k_{max}}$, $k = 1, \ldots, K$. There is also a constraint on the total amount of power that can be allocated in each channel, $P_{n_{max}}^m$, $n = 1, \ldots, N$. This constraint is to control the potential interference to primary users when they return. Assume that SU$_k$ is equipped with $t_k$ antennas at the transmitter and $r_k$ antennas at the receiver, $k = 1, \ldots, K$. Let $H_{ikn}$ denote the MIMO channel from the transmitter of SU$_i$ to the receiver of SU$_k$ on channel
n. The communication channels suffer from large scale fading (path loss/shadowing) and small scale fading due to multipath. The effect of user mobility is not considered in this work and channel gains are assumed to be fixed during channel and power allocation and actual transmissions.

Given the channel conditions and power allocation of all users, the achievable data rate of $S_{U_k}$ is given by [32]

$$R_k = \sum_{n=1}^{N} R_{kn}$$

$$= \sum_{n=1}^{N} \log_2 \left| I + H_{kkn} P_{kn} H_{kkn}^* Q_{kn}^{-1} \right|$$

by treating the interference from other users as additive white Gaussian noise and implementing single user detection. $R_{kn}$ is the achieved data rate on channel $n$. $I$ is an identity matrix of a proper size. $(\cdot)^*$ stands for transpose conjugate. $P_{kn}$ is the transmission power matrix of $S_{U_k}$ on channel $n$. $Q_{kn}$ is noise plus interference from other users to $S_{U_k}$,

$$Q_{kn} = \sigma^2 I + \sum_{i \neq k} H_{ikn} P_{in} H_{ikn}^*$$

$\sigma^2$ is the power of additive white Gaussian noise in each channel. Take eigen-decomposition of symmetric matrix $H_{kkn}^* Q_{kn}^{-1} H_{kkn}$, i.e.,

$$H_{kkn}^* Q_{kn}^{-1} H_{kkn} = V_{kn} \Lambda_{kn} V_{kn}^*$$

where the diagonal elements of $\Lambda_{kn}$, denoted by $\lambda_{kl}$, are the eigenvalues, $l = 1, \ldots, m_k$ and $m_k = \min(t_k, r_k)$. $V_{kn}$ is a unitary matrix containing corresponding eigenvectors. Define $D_{kn} = V_{kn}^* P_{kn} V_{kn}$ in which $P_{kn}$ is chosen such that $D_{kn}$ is a diagonal matrix with diagonal elements denoted by $P_{kl}$. It can be shown that (3.2) can be simplified to [32]

$$R_k = \sum_{n=1}^{N} \sum_{l=1}^{m_k} \log_2 (1 + \lambda_{kl} P_{kl})$$
This indicates that the achievable data rate of SU\textsubscript{k} on channel n is the sum of data rates achieved on \textit{m\textsubscript{k}} equivalent parallel subchannels with channel power gain \textit{\lambda\textsubscript{knl}}. Note that \textit{\lambda\textsubscript{knl}}’s of SU\textsubscript{k} vary with the transmission power of other users.

The problem we are interested in is how to allocate the \textit{N} vacant channels and powers on these channels for the \textit{K} users such that the sum data rate, \( R = \sum_{k=1}^{K} R_k \) is maximized while satisfying the power constraints. The optimization problem to be solved can be expressed as

\[
\max_{\mathbf{P}} \quad R \quad \quad (3.6)
\]

subject to

\[
\sum_{k=1}^{K} \text{tr}(\mathbf{D}_{kn}) \leq P_{\text{mask}}^{n}, \forall n \quad (3.7)
\]

\[
\sum_{n=1}^{N} \text{tr}(\mathbf{D}_{kn}) \leq P_{\text{max}}^{k}, \forall k \quad (3.8)
\]

\[
P_{kl} \geq 0, \forall k, n, l. \quad (3.9)
\]

where \( \mathbf{P} = \{\mathbf{D}_{kn}\}, k = 1, \ldots, K, n = 1, \ldots, N \) is the set of power allocation matrices for all users on all equivalent subchannels. Channel allocation is implicitly embedded in this formulation, i.e., channel \textit{n} is allocated to SU\textsubscript{k} only if \( P_{kl} > 0 \) for any \textit{l}. This optimization problem is generally nonconvex and thus computationally difficult to solve.

Next, a non-cooperative game with pricing is introduced and a distributed iterative algorithm is designed to achieve the maximum sum data rate.
3.2 Channel and Power allocation Algorithm

The common tractable approach is to divide the problem in (3.6)-(3.9) into $K$ subproblems and the $k^{th}$ sub-problem is expressed as

$$\max_{D_k} R_k \quad (3.10)$$

subject to

$$\sum_{k=1}^{K} \text{tr}(D_{kn}) \leq P_{n}^{mask}, \forall n \quad (3.11)$$

$$\sum_{n=1}^{N} \text{tr}(D_{kn}) \leq P_{k}^{max} \quad (3.12)$$

$$P_{knl} \geq 0, \forall n, l. \quad (3.13)$$

where $D_k = [D_{k1}, \ldots, D_{kn}]$. This approach can be interpreted as a non-cooperative game, because each user maximizes its own utility function (which is the data rate itself in this formulation) and ignores the negative impact it has on the network and eventually on itself. This results in an inefficient Nash Equilibrium solution and a suboptimal solution in terms of the sum data rate. In the Game Theory, pricing has been introduced into the utility function to prevent individual users from selfishly transmitting at high power causing unnecessary interference to each other, thus to improve NE solutions. Various types of pricing factors are proposed in the literature. In [31], the authors introduce the path gain as a pricing factor based on heuristic assumptions, whereas in [23] a constant pricing factor found by exhaustive search is used. It is shown in [31] and [23] that, the sum utility is improved by the usage of pricing factor. However, they may still not be Pareto optimal.

Let’s consider the following utility function containing linear pricing functions and a vector of pricing factors $[c_{k1}, \ldots, c_{km_k}]$, which represent the cost of power consumption on each equivalent subchannels,

$$U_k = \sum_{n=1}^{N} \left( R_{kn} - \sum_{l=1}^{m_k} c_{knl} P_{knl} \right) \quad (3.14)$$

Instead of maximizing data rate $R_k$ in (3.10), we maximize $U_k$ in (3.14) for each
user. It will be shown in the sequel that using one pricing factor on total power consumption of SU\(_k\) on channel \(n\), i.e., \(c_{kn} \sum_{l=1}^{m_k} P_{kl}\), cannot lead to a Pareto optimal solution. Eq. (3.14) can be further written as

\[
U_k = \sum_{n=1}^{N} \left( R_{kn} - \text{tr}(C_{kn}D_{kn}) \right)
\]  

(3.15)

where matrix \(C_{kn}\) has diagonal elements \(c_{knl}\), \(l = 1, \ldots, m_k\). \(\text{tr}(\cdot)\) stands for the trace of a matrix. The off-diagonal elements of \(C_{kn}\) do not affect the value of the utility function because \(D_{kn}\) is a diagonal matrix. Our main result is summarized in the following proposition.

**Proposition 1:** Assume that the utility function in (3.15) is adopted in the game of allocating \(N\) channels and power to \(K\) users subject to constraints (3.11)-(3.13). If the resultant NE solution maximizes the sum data rate, then \(C_{kn}\) should take the following form:

\[
C_{kn} = \frac{1}{\ln 2} \sum_{i=1, i \neq k}^{K} V_{kn}^* S_{ikn} V_{kn}
\]  

(3.16)

and

\[
S_{ikn} = H_{kin}^* Q_{in}^{-1} Y_{in}^{-1} H_{in} P_{in} H_{jin}^* Q_{in}^{-1} H_{kin}
\]  

(3.17)

\[
Y_{in} = I + H_{in} P_{in} H_{in}^* Q_{in}^{-1}
\]

\[
Q_{in} = \sigma^2 I + \sum_{j=1, j \neq i}^{K} H_{jin} P_{jn} H_{jin}^*
\]

where \(V_{kn}\) is the unitary matrix defined in (3.4)

**Proof.** Since the optimization problem defined in (3.10)-(3.13) with utility function \(R_k\) replaced by \(U_k\) in (3.15) is concave in \(D_{kn}\), the method of Lagrange multipliers can be used. Let \(\alpha_n, \beta_k\) and \(\gamma_{knl}\) be the Lagrange multipliers. The Lagrangian of the
new optimization problem is given by \[44\]

\[
L_k = \sum_{n=1}^{N} \left( R_{kn} - \text{tr}(C_{kn}D_{kn}) \right) - \sum_{n=1}^{N} \alpha_n \left( \sum_{k=1}^{K} \text{tr}(D_{kn}) - P_{n}^{\text{mask}} \right) \\
- \beta_k \left( \sum_{n=1}^{N} \text{tr}(D_{kn}) - P_{k}^{\text{max}} \right) + \sum_{n=1}^{N} \sum_{l=1}^{m_k} \gamma_{knl} P_{knl}
\]

(3.18)

The solution to maximize \(L_k\) in (3.18) should satisfy the Karush-Kuhn-Tucker (KKT) conditions,

\[
\frac{\partial L_k}{\partial D_{kn}} = \frac{\partial R_{kn}}{\partial D_{kn}} - C_{kn}^T - \alpha_n \mathbf{I} - \beta_k \mathbf{I} + \frac{\partial \left( \sum_{l=1}^{m_k} \gamma_{knl} P_{knl} \right)}{\partial D_{kn}} = 0
\]

(3.19)

\[ P_{knl} \geq 0, \forall n, l \]

\[ \gamma_{knl} P_{knl} = 0, \forall n, l \]

\[ \sum_{n=1}^{N} \text{tr}(D_{kn}) - P_{k}^{\text{mask}} \leq 0 \]

\[ \beta_k \left( \sum_{n=1}^{N} \text{tr}(D_{kn}) - P_{k}^{\text{max}} \right) = 0 \]

\[ \sum_{k=1}^{K} \text{tr}(D_{kn}) - P_{n}^{\text{mask}} \leq 0, \forall n \]

\[ \alpha_n \left( \sum_{k=1}^{K} \text{tr}(D_{kn}) - P_{n}^{\text{mask}} \right) = 0, \forall n \]

(3.20)

On the other hand, for the global optimization problem defined in (3.6)-(3.9), the Lagrangian is given by

\[
L = \sum_{k=1}^{K} \sum_{n=1}^{N} R_{kn} - \sum_{n=1}^{N} \alpha_n \left( \sum_{k=1}^{K} \text{tr}(D_{kn}) - P_{n}^{\text{mask}} \right) \\
- \sum_{k=1}^{K} \beta_k \left( \sum_{n=1}^{N} \text{tr}(D_{kn}) - P_{k}^{\text{max}} \right) + \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{l=1}^{m_k} \gamma_{knl} P_{knl}
\]

(3.21)
The necessary conditions are

\[
\frac{\partial L}{\partial D_{kn}} = \frac{\partial R_{kn}}{\partial D_{kn}} + \sum_{i=1, i \neq k}^{K} \frac{\partial R_{in}}{\partial D_{kn}} - \alpha_n I - \beta_k I + \frac{\partial (\sum_{l=1}^{m_k} \gamma_{knl} P_{knl})}{\partial D_{kn}} = 0 \quad (3.22)
\]

\[P_{knl} \geq 0, \forall k, n \text{ and } l\]

\[\gamma_{knl} P_{knl} = 0, \forall k, n \text{ and } l\]

\[\sum_{n=1}^{N} \text{tr}(D_{kn}) - P_{k}^{\text{max}} \leq 0, \forall k\]

\[\beta_k \left( \sum_{n=1}^{N} \text{tr}(D_{kn}) - P_{k}^{\text{max}} \right) = 0, \forall k\]

\[\sum_{k=1}^{K} \text{tr}(D_{kn}) - P_{n}^{\text{mask}} \leq 0, \forall k \text{ and } n\]

\[\alpha_n \left( \sum_{k=1}^{K} \text{tr}(D_{kn}) - P_{n}^{\text{mask}} \right) = 0, \forall k \text{ and } n\quad (3.23)\]

Comparing the KKT conditions (3.19) and (3.22) of these two problems, we can see that \(C_{kn}\) must satisfy

\[C_{kn}^{T} = - \sum_{i=1, i \neq k}^{K} \frac{\partial R_{in}}{\partial D_{kn}} \quad (3.24)\]

to guarantee that the solution obtained in the first problem maximizes the sum data rate.

As pointed out earlier, only the diagonal elements of \(C_{kn}\) affect the utility function \(U_k\). The diagonal element \(c_{knl}\) is obtained by differentiation with respect to \(P_{knl}\), i.e.,

\[c_{knl} = - \sum_{i=1, i \neq k}^{K} \frac{\partial R_{in}}{\partial P_{knl}}, l = 1, \ldots, m_k\]

It is rare that the diagonal elements are the same. For each \(i\), with \(Y_{in} = I + H_{in} P_{in} H_{in}^{*} Q_{in}^{-1}, Q_{in} = \sigma^2 I + \sum_{j=1, j \neq i}^{K} H_{jin} P_{jn} H_{jin}^{*}, \frac{\partial R_{in}}{\partial P_{knl}}\) can be further expanded
as follows,

\[
\frac{\partial R_{in}}{\partial P_{k^n}} = \frac{1}{\ln 2 |Y_{in}|} \frac{\partial Y_{in}}{\partial P_{k^n}}
\]

\[
= \frac{1}{\ln 2 |Y_{in}|} |Y_{in}| \text{tr} \left( Y_{in}^{-1} \frac{\partial Y_{in}}{\partial P_{k^n}} \right)
\]

\[
= \frac{1}{\ln 2} \text{tr} \left( Y_{in}^{-1} H_{in} P_{in} H_{in}^{\ast} \frac{\partial Q_{in}^{-1}}{\partial P_{k^n}} \right)
\]

\[
= \frac{1}{\ln 2} \text{tr} \left( Y_{in}^{-1} H_{in} P_{in} H_{in}^{\ast} (Q_{in}^{-1}) \frac{\partial Q_{in}^{-1}}{\partial P_{k^n}} Q_{in}^{-1} \right)
\]

\[
= \frac{1}{\ln 2} \text{tr} \left( -V_{kn}^{\ast} H_{kin} Q_{in}^{-1} Y_{in}^{-1} H_{in} P_{in} H_{in}^{\ast} Q_{in}^{-1} H_{kin} V_{kn} \frac{\partial D_{kn}}{\partial P_{k^n}} \right)
\]

\[
(3.25)
\]

Note that matrix \( \frac{\partial D_{kn}}{\partial P_{k^n}} \) is an all-zero matrix except element \((l,l)\) being 1. Any matrix \( X \) multiplying \( \frac{\partial D_{kn}}{\partial P_{k^n}} \) zeros all columns of \( X \) except column \( l \). By taking the trace, element \( X_{(l,l)} \) is the output. If differentiating over the off-diagonal element in \( D_{kn} \), e.g., \( D_{kn}(p,q) \), the output is \( X_{(q,p)} \). Thus, we have

\[
\left( \frac{\partial R_{in}}{\partial D_{kn}} \right)^T = -\frac{1}{\ln 2} V_{kn}^{\ast} S_{ikn} V_{kn}
\]

where \( S_{ikn} = H_{kin}^{\ast} Q_{in}^{-1} Y_{in}^{-1} H_{in} P_{in} H_{in}^{\ast} Q_{in}^{-1} H_{kin} \). From (3.24), we have

\[
C_{kn} = \frac{1}{\ln 2} \sum_{i=1,i \neq k}^{K} V_{kn}^{\ast} S_{ikn} V_{kn}
\]

We complete the proof.

For the special case that single antenna is used, i.e., \( m_k = 1 \) for \( k = 1, \ldots, K \), \( C_{kn} \) reduces to a scalar and \( C_{kn} = \sum_{i \neq k,i=1}^{K} \frac{h_{in} P_{n} h_{kin}}{Q_{in}(Q_{in} + h_{in} P_{in})} \), which matches the result obtained in [6]. From (3.16), we can see that to obtain the optimal set of pricing factors for SU\(_k\) on channel \( n \), besides the unitary matrix \( V_{kn} \) which is available at SU\(_k\), information from others is also required and this is summarized in \( S_{ikn} \), which is a function of SU\(_i\)’s channel state information \( H_{iin} \) and \( H_{kin} \), power allocation matrix
\( \mathbf{P}_{in} \) and interference from other users \( \mathbf{Q}_{in} \). The size of \( \mathbf{S}_{kn} \) \((m_k \times m_k)\) is determined by the number of antennas. \( m_k \) generally takes value 2 or 3 due to the limited size of CR terminals. Thus, in the MAC protocol, all users can communicate this matrix information via control packets with each other to determine their pricing factors. For fixed power allocation of other users, the power that can be allocated on equivalent subchannels for \( \text{SU}_k \) follows a water-filling solution [44], which is given by

\[
P_{knl} = \left[ \frac{1}{\beta_k + c_{knl}} - \frac{1}{\lambda_{knl}} \right] P_{n}^{\text{mask}}
\]

based on Eq. (3.19). \( \beta_k \) is chosen such that the power constraint (3.12) is satisfied. \( P_{knl} \) is limited by the upper bound \( P_{n}^{\text{mask}} \) and the lower bound 0 because of other constraints (3.11) and (3.13). If zero power is allocated on channel \( n \), it implies that \( \text{SU}_k \) does not use that particular channel.

Based on the above analysis, a distributed iterative water-filling algorithm can be designed for channel and power allocation, and it is guaranteed that the resultant NE solution maximizes the sum data rate due to the adaptive pricing in each iteration:

(Initialization): Initialize the transmission powers of all users over all equivalent parallel subchannels to be zero, i.e., \( P_{knl} = 0 \), \( \forall k, \forall n \) and \( \forall l \). Set \( \text{Count} = 0 \).

(IterativeLoop): Repeat the algorithm till either one of the following two conditions is satisfied.

- **Condition 1**: \( \text{Count} \geq \text{Count}_{\text{max}} \), the total number of iterations reaches the maximum value \( \text{Count}_{\text{max}} \).
- **Condition 2**: \( \left| P_{knl}^{(\text{Count})} - P_{knl}^{(\text{Count}-1)} \right| < \epsilon \), \( \forall k, \forall n \) and \( \forall l \). The tolerance \( (\epsilon) \) is generally set to 0.01.

Let \( \text{Count} = \text{Count} + 1 \).

- (UserLoop): Power allocation for each user is conducted sequentially.

For \( k = 1 \) to \( K \),
– (ChannelLoop): Related parameters are evaluated for SU\(_k\) on each channel.

For \(n = 1\) to \(N\),

* Measure the noise plus interference matrix \(Q_{kn}\).
* Calculate the pricing factor matrix \(C_{kn}\) using (3.16) based on information \(S_{ikn}\) gathered from other users, \(i = 1, \ldots, K, i \neq k\).

– Obtain power allocation for SU\(_k\) using (3.26) by selecting a proper \(\beta_k\).

– Send updated \(S_{kn}\) based on (3.17) to SU\(_j\), \(n = 1, \ldots, N, j = 1, \ldots, K, j \neq k\).

### 3.3 Numerical Results

The numerical results are presented to demonstrate the efficiency of the proposed channel and power allocation algorithm. Each SU has two transmitting antennas and two receiving antennas, i.e., \(t_k = r_k = 2, \forall k\). Thus, all channel matrices, \(H_{ikn}\), are of size \(2 \times 2\). Element \((p,q)\) in channel matrix \(H_{ikn}\) is specified as:

\[
h_{ikn}^{(p,q)} = \frac{a_{ik} b_{ikn}^{(p,q)}}{1 + d_{ik}^2}, p, q = 1, 2.
\]

\(d_{ik}\) is the distance between the transmitter of SU\(_i\) and the receiver of SU\(_k\). Shadowing \(a^2_{ik}\) follows the lognormal distribution with mean 1 and \(b_{ikn}^{(p,q)}\) follows the complex Gaussian distribution with zero mean and unit variance. All secondary users can transmit on all the channels, provided they do not violate the \(P_n\) mask constraint and it is 1 Watt. The maximum allowable transmission power for each SU is 2 Watt.

**Example 1.**

The network consists of \(K = 6\) pairs of SUs and there are \(N = 5\) vacant channels. SUs are randomly deployed in a unit area. To illustrate the performance gain of the proposed algorithm, we compare it with the classical iterative water-filling algorithm.
Figure 3.3: Sum data rates of IWF and OIWF. $K = 6$, $N = 5$ and $P_{k}^{\text{max}} = 2W$. (IWF) without pricing. Since our algorithm achieves the maximum sum data rate, we denote it by OIWF. Both iterative algorithms reach Nash Equilibria within 4-7 steps. The resultant sum data rate as a function of noise power levels is plotted in Fig. 3.3. The sum data rates decrease as the noise power increases. When the noise power is low, the achieved sum data rate by OIWF is much larger than that of IWF. This is because in IWF, each user tries to maximize its own data rate by allocating maximum allowable power on all subchannels, causing high interference to other users. This adverse effect eventually impacts the data rate of itself, thus lowers the sum data rate. Whereas in OIWF, a variable power level due to the adaptive pricing factors is used on each subchannel as it imposes a punishment to the power consumption of each user proportional to the interference it causes to other users. This results in low interference to other users and higher sum data rate. When the noise power is large, additive channel noise becomes the dominant factor to the achievable data rate. The difference between the achieved sum data rates of IWF and OIWF with adaptive pricing becomes small. Under the same conditions of channels and $P_{n}^{\text{mask}}$, but with
\(P_{k}^{\text{max}} = 1\) Watt, the resultant sum data rates of IWF and OIWF are depicted in Fig. 3.4. Comparing Fig. 3.3 and Fig. 3.4, we observe that as the maximum allowable transmission power for each user decreases, the achievable sum data rate decreases.

![Figure 3.4: Sum data rates of IWF and OIWF.](image)

**Figure 3.4:** Sum data rates of IWF and OIWF. \(K = 6, N = 5\) and \(P_{k}^{\text{max}} = 1\)W.

**Example 2.**

To demonstrate the optimality of using a vector of pricing factors for each user on each channel, we compare OIWF to a scenario where each user employs only one pricing factor per channel to penalize the total power consumption of each user on that particular channel. The approach in [23] is adopted, i.e., the constant pricing factor per channel per user is found by exhaustive search (ES). We set \(K = 2\) and \(N = 3\). Note that our proposed distributed iterative algorithm can be applied to any network size and the number of available channels. In Fig. 3.5, the sum data rates obtained using IWF, OIWF and the IWF algorithm with one constant pricing factor per user per channel obtained via ES are plotted. Adopting pricing in the game indeed improves NE solutions, comparing the performances of IWF. However, this intuitive
pricing formulation using ES still cannot guarantee Pareto optimal solutions. By using different pricing factors for each user on each subchannel, the best performance in terms of sum data rate can be achieved. The power allocations satisfy the power mask constraint of 0.5 Watt and the maximum allowable power of 1 Watt and are plotted in Fig. 3.6, Fig. 3.7 and Fig. 3.8.

Figure 3.5: Sum data rates of IWF, OIWF and ES. $K = 2$ and $N = 3$.

Figure 3.6: Power allocations for IWF.
Figure 3.7: Power allocations for OIWF.

Figure 3.8: Power allocations for ES.
Example 3.

We simulate a network consisting of 2 pairs of SUs and 3 vacant channels under two channel conditions, one is a strong interference scenario and the other is a weak interference scenario. In the strong interference scenario, the users are randomly deployed in the unit area so that the potential interference to each other is stronger, shown in Fig. 3.9. In the weak interference scenario, the 2 pairs of SUs are placed far apart so that the interference to each other is weak, shown in Fig. 3.13. In IWF, each user allocates the total available power on all channels, irrespective of the channel conditions. So, if the channel conditions are poor (strong interference scenario), each user will cause high interference to other users. This negative effect reduces the system efficiency and eventually its performance. Therefore, the IWF performance degrades in strong interference scenario compared to weak interference scenario. Whereas in OIWF, a variable power level is adopted due to the pricing factor, signifying the punishment to each user proportional to the interference it causes to other users. This results in less interference to other users and a higher sum data rate. Thus from Fig. 3.10 and Fig. 3.14, we observe that the performance gain of OIWF over IWF is more in Fig. 3.10 than in Fig. 3.14. The power allocations satisfy the power mask constraint of 0.5 Watt and the maximum allowable power of 1 Watt and are plotted in the following figures.
Figure 3.9: CR network for strong interference scenario.

Figure 3.10: Sum data rates of IWF and OIWF for strong interference. $K = 2$ and $N = 3$. 
Figure 3.11: Power allocations for IWF in strong interference scenario.

Figure 3.12: Power allocations for OIWF in strong interference scenario.
Figure 3.13: CR network for weak interference scenario.

Figure 3.14: Sum data rates of IWF and OIWF for weak interference. $K = 2$ and $N = 3$. 
Figure 3.15: Power allocations for IWF in weak interference scenario.

Figure 3.16: Power allocations for OIWF in weak interference scenario.
CHAPTER 4

Joint Channel and Power Allocation based on User Satisfaction

In this chapter, a new metric indicating user satisfaction in both data rate and power consumption is formulated. Joint channel and power allocation algorithms are designed for both orthogonal transmission and interference transmission for cognitive radios with single antenna. These algorithms are then extended for cognitive radios equipped with multiple antennas. Numerical results are presented to demonstrate the efficiency of the proposed algorithms. The performance comparison is conducted for orthogonal and interference transmissions under different interference scenarios. The performance gains by allowing interference transmission and by using multiple antennas at transceivers are demonstrated.

4.1 Problem Formulation

Consider a wireless network with $N$ channels and $K$ SU’s. The total power constraint on each user is $P_{k}^{\text{max}}, k = 1, \ldots, K$. There is also a power mask constraint, $P_{n}^{\text{mask}}, n = 1, \ldots, N$ in each channel that prevents the SU’s from transmitting with high power so as to control the interference to PU’s when they return. Multiple SU transmissions are allowed in a channel at the same time provided they do not violate the $P_{n}^{\text{mask}}$ constraint. Due to multipath, the channels between CRs suffer from large scale fading and small scale fading and are assumed to be constant during the evaluation of the algorithm.

As discussed in Chapter 2, many utility functions can be formulated as different users can have different requirements on data rates depending on the type of services.
One such utility function is user satisfaction and it increases when approaching the target data rate. However, exceeding the desired target data rate is unnecessary and even harmful in terms of causing extra interference to other users. For mobile users powered by onboard battery, power efficiency is a very important factor. The less is power consumption for transmission, the higher is user satisfaction. Our proposed metric which is defined as a weighted sum of data rate satisfaction and power consumption satisfaction, takes into account the desired quality of service (QoS) in terms of the target data rate and transmission power constraints in an explicit way. This is different from existing price-based utility formulation [35], which maximizes the data rate neglecting the user’s satisfaction. With the objective function defined, the goal is to maximize total user satisfaction. We assume SU$_k$ has an expected target data rate, $R^\text{tar}_k$, for the type of service requested, e.g., text messages, video. If the achieved data rate $R_k$ is less than $R^\text{tar}_k$, the user is less satisfied. If $R_k \geq R^\text{tar}_k$, the user is fully satisfied in terms of the data rate. On the other hand, the less power ($P_k$) is consumed, the higher is user satisfaction. Based on these, SU$_k$’s satisfaction is defined as

$$S_k = \gamma_k f(R_k, R^\text{tar}_k) + (1 - \gamma_k) g(P_k, P^\text{max}_k)$$  \hspace{1cm} (4.1)

where $f(R_k, R^\text{tar}_k)$ is data rate satisfaction and $g(P_k, P^\text{max}_k)$ is power consumption satisfaction. For example, we can select $f(\cdot)$ as shown in Fig. 4.1, i.e.,

$$f(R_k, R^\text{tar}_k) = \begin{cases} 
1 & R_k \geq R^\text{tar}_k \\
\frac{R_k}{R^\text{tar}_k} & 0 \leq R_k \leq R^\text{tar}_k
\end{cases}$$  \hspace{1cm} (4.2)

and $g(\cdot)$ as shown in Fig. 4.2, i.e.,

$$g(P_k, P^\text{max}_k) = 1 - \frac{P_k}{P^\text{max}_k}, 0 \leq P_k \leq P^\text{max}_k$$  \hspace{1cm} (4.3)

Other types of satisfaction functions can also be designed for different applications. (4.2) and (4.3) will be adopted for subsequent analysis. $\gamma_k$ is a user-specific
Figure 4.1: Data rate satisfaction

Figure 4.2: Power consumption satisfaction
tuning parameter for adjusting the weights between data rate and power consumption satisfactions. Small $\gamma_k$ implies user’s preference of saving its battery while accepting a lower data rate whereas large $\gamma_k$ means SU$_k$ prefers giving up a long battery lifetime to fulfill its data rate needs.

By assuming all SUs are equally important, total user satisfaction is given by $S = \sum_{k=1}^{K} S_k$. It is easy to incorporate users with different levels of service. Obviously, total user satisfaction is a function of the power allocation matrix $\mathbf{P} = \{P_{kn}\}$. Channel allocation is included implicitly, i.e., channel $n$ is allocated to SU$_k$ only if $P_{kn} > 0$. The problem becomes, after taking into account the channel state information, how to allocate the $N$ vacant channels and powers on these channels such that $S$ is maximized and meanwhile all power constraints are satisfied.

\[
\max_{\mathbf{P}} \quad S(\mathbf{P}) \\
\text{subject to} \quad \sum_{k=1}^{K} P_{kn} \leq P_{n}^{\text{mask}}, \quad \sum_{n=1}^{N} P_{kn} \leq P_{k}^{\text{max}}, \quad P_{kn} \geq 0, \forall k, n. \tag{4.4}
\]

As discussed before, there are two distinct ways to allocate channels: 1) orthogonal transmission, one channel can only be occupied by one user, and 2) interference transmission, one channel can be used by several users. Each user can also be equipped with a single antenna or multiple antennas with increased complexity. Next we discuss the design for each of these configurations separately.

### 4.2 Joint Channel and Power Allocation for Orthogonal Transmission

In orthogonal transmission, there is no interference between SUs because they occupy different channels. This is essentially frequency division multiplexing. In this setting, we can define the channel allocation matrix $\mathbf{A} = \{\alpha_{kn}\}$ explicitly. $\alpha_{kn} = 1$ if channel
n is assigned to SU$_k$ and 0 otherwise. Constraint $\sum_{k=1}^{K} \alpha_{kn} = 1$ highlights that each channel can be assigned to only one SU. Only channel state information for intended transmission needs to be considered in this setting.

4.2.1 Single Antenna

Given the channel allocation matrix $A$ and the power allocation matrix $P$, the achieved data rate of SU$_k$ is

$$R_k = \sum_{n=1}^{N} \alpha_{kn} \log \left( 1 + \frac{|h_{kn}|^2 P_{kn}}{\sigma^2} \right)$$

where $h_{kn}$ is the channel gain of SU$_k$ on channel $n$, $\sigma^2$ is the power of additive white Gaussian noise and is assumed the same across all channels. $P_{kn} = 0$ if $\alpha_{kn} = 0$.

The optimization over $P = [P_1, \ldots, P_K]$ in (4.4) can be rewritten as

$$\max_A \left( \sum_{k=1}^{K} \max_{P_k} S_k \right),$$

where $S_k = \gamma_k \min \left( 1, \frac{R_k}{R_{\text{tar}}^k} \right) + (1-\gamma_k) \left( 1 - \sum_{\{n: \alpha_{kn} = 1\}} \frac{P_{kn}}{P_{\text{max}}^k} \right)$.

If $A$ is given, then (4.4) can be separated into $K$ subproblems. In other words, for each user, power allocation on the assigned channels needs to be optimized to maximize his own satisfaction,

$$\max_{P_k} S_k$$

with constraints $0 \leq P_{kn} \leq P_{\text{mask}}^n$ and $\sum_{\{n: \alpha_{kn} = 1\}} P_{kn} \leq P_{\text{max}}^k$. It can be shown that the optimization in (4.9) leads to the well-known water-filling solution [45]. However, the power level cannot be simply chosen to satisfy power constraints. Using more power can increase achieved data rates, yet it may not be in the best of users’ interest considering the increased power consumption. The following two propositions provide the solution of the optimal power level.

**Proposition 1**: If the condition holds that when $P_{\text{max}}^k$ is used, the achievable data rate $R_k \leq R_{\text{tar}}^k$, then the optimal power level is given by

$$L_k^o = \frac{\gamma_k P_{\text{max}}^k}{(1 - \gamma_k) R_{\text{tar}}^k \ln 2}$$

(4.10)
Proof. Given the power level $L_k^o$, the power allocated on the assigned channels can be expressed as $P_{kn} = \min \left( \max \left( L_k^o - \frac{\sigma^2}{|h_{kn}|^2}, 0 \right), P_n^{mask} \right)$. Assume that there are $x$ channels with allocated power $P_{kn} = L_k^o - \frac{\sigma^2}{|h_{kn}|^2}$ and $y$ channels with allocated power $P_n^{mask}$. Then SU$_k$’s satisfaction is given by

$$S_k = \frac{\gamma_k}{R_k^{tar}} \left[ \sum_{\{x\}} \log \left( 1 + \frac{\left( L_k^o - \frac{\sigma^2}{|h_{kn}|^2} \right) |h_{kn}|^2}{\sigma^2} \right) + \sum_{\{y\}} \log \left( 1 + \frac{|h_{kn}|^2 P_n^{mask}}{\sigma^2} \right) \right]$$

$$+ (1 - \gamma_k) \left[ 1 - \frac{\sum_{\{x\}} \left( L_k^o - \frac{\sigma^2}{|h_{kn}|^2} \right) + y P_n^{mask}}{P_k^{max}} \right]$$

$$= \frac{\gamma_k}{R_k^{tar}} \left[ \sum_{\{x\}} \log \left( \frac{|h_{kn}|^2}{\sigma^2} \right) + x \log(L_k^o) + \sum_{\{y\}} \log \left( 1 + \frac{|h_{kn}|^2 P_n^{mask}}{\sigma^2} \right) \right]$$

$$+ (1 - \gamma_k) \left[ 1 - \frac{x L_k^o}{P_k^{max}} + \frac{\sum_{\{x\}} \left( \frac{\sigma^2}{|h_{kn}|^2} - y P_n^{mask} \right)}{P_k^{max}} \right]$$

Taking a derivative of (4.11) over $L_k^o$, we have

$$\frac{\partial S_k}{\partial L_k^o} = \frac{\gamma_k x}{R_k^{tar} L_k^o \ln 2} - \frac{(1 - \gamma_k) x}{P_k^{max}}$$

(4.11)

By setting (4.14) to zero, we have $L_k^o = \frac{\gamma_k P_k^{max}}{(1 - \gamma_k) R_k^{tar} \ln 2}$. Since $\frac{\partial^2 S_k}{\partial (L_k^o)^2} < 0$, $L_k^o$ maximizes $S_k$ if SU$_k$ cannot achieve $R_k^{tar}$ even using all available power $P_k^{max}$.

This result implies that if the target data rate is not achievable even when all power is allocated, then the power level should be set at $L_k^o$ to maximize user satisfaction. We observe that this power level is independent of the noise floor. When the maximum available power increases and the target data rate is reduced, the power level should be raised, which matches our intuition.

Proposition 2: There are three power levels, 1) $L_k^P$ when allocating all power $P_k^{max}$, 2) $L_k^R$ that achieves $R_k^{tar}$, and 3) $L_k^o$ from Proposition 1 that maximizes SU$_k$’s satisfaction under the condition that the achievable data rate is no greater than $R_k^{tar}$ and the total power allocated is no greater than $P_k^{max}$. The optimal power level satisfying all constraints is $L_k = \min \left( [L_k^P, L_k^R, L_k^o] \right)$.
Proof. $L_k^o$ is optimal under the assumption that power consumption $P_k^o$ and the achieved data rate $R_k^o$ with this power level are no greater than $P_{k, max}^o$ and $R_{k, tar}^o$, respectively. If resulted $P_k^o > P_{k, max}^o$, then it is obvious that $L_k^o > L_k^p$ and $L_k^p$ should be chosen to satisfy the power constraint. If, on the other hand, $R_k^o > R_{k, tar}^o$, then $P_k^o$ is not optimal because we can always find a reduced $P_k^R$ which achieves exact $R_k^R = R_{k, tar}^R$ with higher user satisfaction. Thus, the optimal power level satisfying all power constraints is the smallest one among these three power levels.

The power allocation for SU$k$ is then given by $P_{kn} = \min \left( \max \left( L_k - \frac{\sigma^2}{|h_{kn}|^2}, 0 \right), P_{n, mask} \right)$. Note that the determination of the power level here is different from chapter 3 when the objective is to maximize data rates.

Total user satisfaction can be further improved by allocating channels optimally. For $K$ users and $N$ channels, there are total $K^N$ possible channel allocation schemes. Exhaustive search over all these possibilities to find the optimal scheme is practically infeasible. Here, we propose a channel-by-channel optimization (CBCO) method [46] in which the best allocation of each channel is derived by fixing the allocation of the rest channels iteratively.

1. Initialization: Set $l = 0$, the channel allocation matrix $A^l$ is initialized. The power allocation matrix $P^l$ is found to maximize each user satisfaction based on the two propositions above. The achieved total user satisfaction is denoted by $S^l$.

2. Iteration: $l = l + 1$. For channel $n = 1$ to $N$, allocate channel $n$ to SU$k$, $k = 1, \ldots, K$ that maximizes total user satisfaction. After allocating channel $N$, the achieved $S^l$ is compared to $S^{l-1}$. If $S^l - S^{l-1} < \epsilon$, stop and accept both $A^l$ and $P^l$ as the final solution for joint channel and power allocation. Otherwise, go back to step 2).

Since $S^l$ is non-decreasing at each iteration, the convergence of the algorithm is
guaranteed. However, this greedy approach may result in local optima. We observe that if all users have the same configuration, then they have the same power level (4.10) for power allocation if all constraints are met. Intuitively, if a channel is allocated to the user with the smallest noise floor, then largest contribution can be obtained towards total user satisfaction. Based on this, instead of random initialization, we can allocate each channel to the user with the smallest noise floor at the initial step. Fig. 4.3 shows total user satisfaction by using exhaustive search and CBCO with both the proposed nonrandom and random initializations. CBCO with nonrandom initialization can achieve a performance close to that of exhaustive search while the computational complexity of this method is only $O(KN)$ instead of $O(K^N)$. This is also observed for nonidentical user configurations.

![Figure 4.3: Total user satisfaction for orthogonal transmission with single antenna.](image)

### 4.2.2 Multiple Antennas

With multiple antennas at each transceiver, we deal with vector channels instead of scalar channels, $\mathbf{H}_{kn}$ with dimension $r_k \times t_k$, where $t_k$ is the number of transmitting
antennas and $r_k$ is the number of receiving antennas of SU$_k$. $h_{kn}^{ij}$ is the channel gain between transmitting antenna $i$ and receiving antenna $j$ of SU$_k$ on channel $n$. We not only need to find for SU$_k$ the optimal power allocation on each channel, but also power allocation on each antenna to maximize user satisfaction. If we apply singular value decomposition on $H_{kn}$, i.e., $H_{kn} = U \Lambda V$, then the power allocation on the $t_k$ antennas is equivalent to power allocation on $m_k = \min(t_k, r_k)$ parallel subchannels, the channel gains of which are the diagonal elements of $\Lambda, \lambda_l, l = 1, \ldots, m_k$. The data rate of SU$_k$ can be expressed as

$$R_k = \sum_{n=1}^{N} \alpha_{kn} \sum_{l=1}^{m_k} \log\left(1 + \frac{\lambda_l^2 P_{knl}}{\sigma^2}\right)$$

(4.12)

$P_{knl} = 0$ if $\alpha_{kn} = 0$. Note that $P_{knl}$ here is the power allocated on equivalent sub-channel $l$, not the true power allocated on transmitting antennas. The joint channel and power allocation algorithm proposed for the single-antenna configuration can be adopted. Note that each channel here consists of several equivalent parallel subchannels depending on the number of antennas.

Assume that there are $U$ channels with allocated power $P_{kul} = L_o^k - \frac{\sigma^2}{\lambda^2_{ul}}$ on each parallel subchannel and $V$ channels with allocated power $P_v^{mask}$. Since this is a MIMO system, the power mask constraint results in $\sum_{l=1}^{m_k} P_{kul} \leq P_v^{mask}$. Let $L_v^{mask}$ be the power level on each subchannel of channel $V$, then the power allocated on each subchannel of channel $V$ is $P_{kv} = L_v^{mask} - \frac{\sigma^2}{\lambda^2_{vl}}$. Then user $k$’s satisfaction is given by

$$S_k = \frac{\gamma_k}{R_{k_{tar}}} \left[ \sum_{\{U\}} \sum_{l=1}^{m_k} \log\left(1 + \frac{(L_o^k - \frac{\sigma^2}{\lambda^2_{ul}}) \lambda_l^2}{\sigma^2}\right) + \sum_{\{V\}} \sum_{l=1}^{m_k} \log\left(1 + \frac{(L_v^{mask} - \frac{\sigma^2}{\lambda^2_{vl}}) \lambda_l^2}{\sigma^2}\right) \right]$$

$$+ (1 - \gamma_k) \left[ 1 - \frac{\sum_{\{U\}} \sum_{l=1}^{m_k} (L_o^k - \frac{\sigma^2}{\lambda^2_{ul}}) + V P_v^{mask}}{P_{k_{max}}} \right]$$

$$= \frac{\gamma_k}{R_{k_{tar}}} \left[ \sum_{\{U\}} \sum_{l=1}^{m_k} \log\left(\frac{\lambda^2_{ul}}{\sigma^2}\right) + U m_k \log(L_o^k) + \sum_{\{V\}} \sum_{l=1}^{m_k} \log\left(1 + \frac{(L_v^{mask} - \frac{\sigma^2}{\lambda^2_{vl}}) \lambda_l^2}{\sigma^2}\right) \right]$$

$$+ (1 - \gamma_k) \left[ 1 - \frac{U m_k L_o^k}{P_{k_{max}}^{mask}} + \frac{\sum_{\{U\}} \sum_{l=1}^{m_k} (\frac{\sigma^2}{\lambda^2_{ul}}) - V P_v^{mask}}{P_{k_{max}}^{mask}} \right]$$

(4.13)
Taking a derivative of (4.13) over \( L^o_k \), we have

\[
\frac{\partial S_k}{\partial L^o_k} = \frac{\gamma_k U m_k}{R^\text{tar}_k L^o_k \ln 2} - \frac{(1 - \gamma_k) U m_k}{P^\text{max}_k} \tag{4.14}
\]

By setting (4.14) to zero, we have \( L^o_k = \frac{\gamma_k P^\text{max}_k}{(1 - \gamma_k) R^\text{tar}_k \ln 2} \). Since \( \frac{\partial^2 S_k}{\partial (L^o_k)^2} < 0 \), \( L^o_k \) maximizes \( S_k \) if SU\(_k\) cannot achieve \( R^\text{tar}_k \) even using all available power \( P^\text{max}_k \).

### 4.3 Power Allocation for Interference Transmission

If we allow several SUs transmitting on the same channels, it inevitably causes interference to each other, the magnitude of which depends on many factors. Each user is assumed to implement single user detection by treating interference from other users as additive white Gaussian noise for simplicity. Different from the problem formulated under orthogonal transmission, there is no explicit optimization over channel allocation. Channel selection is embedded in power allocation, i.e., little or zero power allocation on a channel indicates not using that particular channel. The problem that needs to be solved is in the form of (4.4)-(4.7). Similar to Section 4.2, there are also single-antenna and multiple-antenna configurations which are discussed separately next.

#### 4.3.1 Single Antenna

Let \( h_{ikn} \) denote the channel gain between SU\(_i\)’s transmitter and SU\(_k\)’s receiver on channel \( n \). Due to the interference from other SUs, the data rate of SU\(_k\) is given by

\[
R_k = \sum_{n=1}^{N} \log(1 + \frac{|h_{kn}|^2 P_{kn}}{\sigma^2 + \sum_{i \neq k} |h_{ikn}|^2 P_{in}}) \tag{4.15}
\]

The optimization of power allocation to maximize total user satisfaction is in general nonconvex and is hence computationally difficult to solve. Therefore, we are motivated to design tractable suboptimal algorithms. Given the power allocation of all other SUs, the optimization of power allocation for SU\(_k\) can be carried out based on
the method proposed in Section 4.2.1. The only difference is that now the noise floor, instead of being constant, is a function of transmission power of other users. The update in power allocation of one SU will cause a change in interference to other SUs. Thus, other SUs need to update their power allocation accordingly. If the process continues iteratively, Nash Equilibrium will be achieved [35]. However, NE is not optimal in terms of total user satisfaction. Similar to the CBO procedure, new power allocation for each user should be accepted only if there is an increase in total user satisfaction. This guarantees that the objective is nondecreasing at each iteration. The drawback is that this may lead to local optima. Power allocation initialization becomes important in this sense. Both zero initialization where zero power is allocated for all SUs across all channels [3] [6] and uniform initialization where $\frac{P_{\text{max}}}{N}$ is allocated to all channels for SU$_k$ at the initial step [35] are proposed in literature. Here we propose to initialize the iterative algorithm with optimal power allocation for orthogonal transmission. Intuitively, interference transmission should perform at least as well as orthogonal transmission because of the relaxed channel allocation constraint. Fig. 4.4 compares the performance of different initialization schemes. Initialization with orthogonal allocation results outperforms zero and uniform initializations.

4.3.2 Multiple Antennas

Let $H_{ikn}$ denote the interference channel gain to SU$_k$ from other users, $i \neq k$. The dimension of $H_{ikn}$ is $r_k \times t_i$. By adopting single user detection, the data rate can be expressed as

$$R_k = \sum_{n=1}^{N} \log_2 \left( |I + H_{kkn} P_{kn} H_{kkn}^* Q_{kn}^{-1}| \right)$$

where $Q_{kn} = \sigma^2 I + \sum_{i \neq k} H_{ikn} P_{in} H_{ikn}^*$. The same iterative algorithm and initialization scheme as for multiple-antenna in orthogonal transmission can be adopted. To optimize power allocation for each SU, equivalent parallel subchannels are obtained first by SVD. Note that these equivalent subchannels vary each time when the
4.4 Numerical Results

To evaluate the efficiency of proposed algorithms, we simulate $K = 4$ SUs randomly deployed in a unit area. There are $N = 7$ vacant channels available. Let $d_{ij}$ denote the distance between SU$_i$’s transmitter and SU$_j$’s receiver. The channel gain is represented by $h_{ijn} = \frac{a_{ij}b_{ijn}}{1+d_{ij}^2}$. Shadowing $a_{ij}^2$ follows the lognormal distribution with mean 1 and $b_{ijn}$ follows the complex Gaussian distribution with zero mean and unit variance. Other parameters are specified as follows: $P^\text{mask}_n = 0.5$, $P^\text{max}_k = 2$, $R^\text{tar}_k = 8$, $\gamma_k = 0.5$ and $t_k = r_k = 2$. Note that $\sum_{k=1}^{K}(1 - \gamma_k) \leq S < K$. The lower bound is achievable when no power is allocated.

In the first experiment, we deliberately put 4 pairs of cognitive radios far apart so that the interference, if any, to each other is weak. As shown in Fig. 4.5, each user consists of one transmitter and one receiver. The performances of 4 different configurations (orthogonal/interference transmission with single/multiple antennas)
Figure 4.5: CR network for weak interference scenario.

Figure 4.6: Comparison of total user satisfaction with weak interference.
by using the proposed algorithms are compared in Fig. 4.6. Total user satisfaction is a decreasing function of noise power. When noise power is large and the target data rate is difficult to achieve, all SUs tend to use very little power thus total user satisfaction is close to 2. On the other hand, when noise power is very low, reaching the target data rate costs only very little power, thus total user satisfaction is close to 4. The performance gain by using two antennas at each transceiver is significant compared to the single-antenna case. Interference transmission outperforms orthogonal transmission and the performance gain is larger for multiple-antenna than that for single-antenna. This is because multiple antennas provide extra spatial diversity to combat multiuser interference.

![Figure 4.7: CR network for strong interference scenario.](image)

In the second experiment, the users are randomly deployed in the region thus the potential interference to each other is stronger, shown in Fig. 4.7. The same four configurations are compared in Fig. 4.8. The performance gain by allowing interference transmission is reduced. These results indicate that when interference is weak, frequency reuse is encouraged by allowing more interference transmission, and it should be avoided if potential interference is strong.
Figure 4.8: Comparison of total user satisfaction with stronger interference.
CHAPTER 5

Conclusions

In this work, we consider the problem of joint channel and power allocation in a network consisting of $K$ cognitive radio users and $N$ vacant channels. Upon extensive literature review, we show that there are two distinct ways to allocate channels. One is restricted to orthogonal transmission and the other allows interference transmission. Cognitive radios can be equipped with a single antenna or multiple antennas with increased complexity.

Using the game theory, we design two different objective functions based on user requirements and formulate different optimization problems for the aforementioned settings, i.e., orthogonal transmission v.s. interference transmission, single antenna v.s. multiple antennas. In Chapter 3, we consider the problem of maximizing the sum data rate, with a set of power constraints and achieve the joint channel and power allocation for cognitive radio users equipped with multiple antennas at both transmitters and receivers. By employing a non-cooperative game with pricing formulation, we derive the formula of a vector of adaptive pricing factors for each user on each channel and prove that the resultant NE solution of the iterative water-filling algorithm achieves the maximum sum data rate. Our method not only makes the complex global optimization problem tractable, but also facilitates distributed implementation. A distributed algorithm is designed where only small amount of information exchange between users is required in each iteration for practical CR users. Numerical results demonstrate the effectiveness of the proposed algorithm compared to the classical iterative water-filling algorithm without pricing and other intuitive
pricing formulations.

In Chapter 4, we introduce a new metric for total user satisfaction and consider the problem of maximizing total user satisfaction, with a set of power constraints. First, we solve for the joint channel and power allocation for cognitive radio users equipped with single antenna at both transmitters and receivers for both, orthogonal transmission and interference transmission and then extend it to include the multiple-antenna configuration. For the orthogonal transmission, we have an explicit algorithm for channel allocation whereas in interference transmission the channel allocation is embedded in power allocation. The efficiency of the proposed algorithms and performance comparisons are demonstrated through simulations.
BIBLIOGRAPHY


[26] C. Shi, R. Berry, and M. Honig, “Distributed interference pricing for OFDM wire-
less networks with non-separable utilities,” *Information Sciences and Systems*,

with cognitive radios: An information theoretic perspective,” *IEEE J. Select.

Radio System,” *IEEE Transactions on Communications*, vol. 57, no. 7, pp. 1928-
1931, July 2009.


*Wireless Communications, Networking and Mobile Computing*, pp. 1256 - 1259,
Sept. 2007.


[33] R. Yellapantula, Y. Yingwei anf R. Ansari, “Antenna selection and power control
in MIMO systems with continuously varying channels,” *IEEE Commun. Letters*,

[34] Y. Song and S. D. Blostein, “MIMO channel capacity in co-channel interference,”
*Proc 21st Biennial Symposium on Communications*, Kingston, Canada, pp. 220-
224, Jan. 2002.


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The concept of cognitive radio with many promising features like spectrum sensing, dynamic spectrum access has the potential to greatly improve the spectral efficiency. In this thesis, we study the problem of spectrum-hole sharing, i.e., vacant channel and power allocation to multiple cognitive radio (CR) users who coexist in a network. Based on the game theory, we formulate two different objective functions considering user requirements. In the first scenario, the sum data rate of a network is maximized. A non-cooperative game with pricing is formulated for interference transmission in which each cognitive radio user is equipped with multiple antennas at both transmitter and receiver. Each user incorporates a vector of adaptive pricing factors on each channel based on transmission power of other users. The formula of optimal pricing is derived and a distributed iterative algorithm is designed through small amount of information exchange between users. The simulation results demonstrate the performance gain of the proposed algorithm over the classical iterative water-filling algorithm without pricing and intuitive pricing formulations.

In the second scenario, a new metric indicating user satisfaction in both data rate and power consumption is formulated. Initially, joint channel and power allocation algorithms are designed for both orthogonal transmission and interference transmission for cognitive radio users equipped with single antennas. Then, these algorithms are extended for multiple antennas configuration. Numerical results demonstrate the efficiency of the proposed algorithms. The performance comparison is conducted for orthogonal and interference transmissions under different interference scenarios. The performance gains by allowing interference transmission and by using multiple antennas at transceivers are demonstrated.