PRELIMINARY AND FINAL DESIGN OF A DIRECT-CURRENT MAGNETIZING COIL

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A direct-current magnetizing coil is a substantial part or element of most electrical machines and apparatus. It is exemplified as mounted on an electromagnet in Figs. 1 to 4 showing openings of rectangular, circular, and other forms. Usually the conditions or requirements which should be complied with or fulfilled by a designer are as follows.

1. A terminal voltage $V$; it may be determined by line voltage or otherwise.

2. A fixed number of ampere-turns ($n i$) where $n$ is the number of winding turns and $i$ the current in the winding.

3. A surface temperature rise $T_r$; in other words, an outside rise—a difference between the surface temperature of the coil and the ambient temperature.

4. Also, very often, a specific size and shape of coil opening.
5. Sometimes special requirements as to length and/or width of the winding space; we call them simply the length and width, or thickness, of the coil.

6. \( V \) is given as prescribed by the current source; \((ni)\) and also the size and shape of the coil opening are usually predetermined on the basis of the magnetic-circuit specification and performance of the electrical machine or apparatus; the temperature rise \( T_r \) is mostly chosen within limits depending on the ambient temperature and restrictions imposed by insulating materials at the hottest spot inside the coil, or, in other words, an appropriate temperature \( T_h \) there is assumed.

There are given: number of ampere-turns \((ni)\); outside temperature rise \( T_r \); terminal voltage \( V \); size and form of coil opening, \( p \) being the length of its perimeter.

There are sought: size of conductor, i.e., its cross-section area \( q \); area of winding space indicated by \( l \) and \( w \), its length and width respectively, which are at the same time the length and thickness of the coil; number of ampere-turns \( n \).

The derivations are brought about by using auxiliary formulas, as follows.

The electrical losses or generated heat in watts:

\[
W = i^2 R = V^2 / R
\]

The length of the mean turn:

\[
m = p + kw
\]

where the value of \( k \) depends upon the form of the opening. For openings of circular form (Fig. 2), of rectangular shape with perfectly rounded corners, and of the type represented in Fig. 4, \( k = \pi \); for the rectangular opening with square corners (Fig. 3) \( k = 4 \); for the rectangular opening having corners not perfectly rounded \( \pi < k < 4 \)—the first plausible assumption in this case would be averagely to make \( k = 3.57 \).

The cooling surface:

\[
A = 2mw + pl + (p + 2kw)l = 2(w + l) (p + kw) = 2m(w + l)
\]

The outside temperature rise:

\[
T_r = W / cA
\]

The total winding resistance:

\[
R = \rho n (p + kw) / q
\]

where \( \rho \) designates resistivity at the average temperature of the coil wire.

The number of winding turns:

\[
n = stw / q
\]

where \( s \) is the space factor which is computable or available from graphs if \( q \) and the mode of insulation of the conductors are known or determinable.

The current in the coil winding:

\[
i = Vq / qn(p + kw)
\]

Further derivations are as follows.

Because of (7)

\[
(ni) = Vq / q(p + kw)
\]
Referring to (4)
\[ W = cT \tau A \] ................................. (8).

Again, because of (1),
\[ \nu R = cT \tau A \] ................................. (9).

Referring to (2) and (5)
\[ R = qnm/q \] ................................. (10).

Then from (9) and (10)
\[ \nu R = \nu qnm/q = \nu (nt) qm/q = cT \tau A \] ................................. (11).

Again, denoting current density by \( \Delta \), it obtains that
\[ i = q \Delta, \Delta = (nt)/swl, \text{ and } i = q(nt)/swl \] ................................. (12).

Substituting (12) for \( i \) and (3) for \( A \) we derive from (11) the fundamental equation:
\[ q(ni)^2/swl = 2cT \tau (w+1) \] ................................. II.

Then
\[ q(ni)^2/2swcT \tau = lw + 1^2 \] ................................. (13).

Let
\[ Q_W = q(ni)^2/2swcT \tau \] ................................. III.

Now (13) transforms to the quadratic equation:
\[ lw^2 - Q_W = 0 \] ................................. IV.

From this it follows that
\[ l = -w^2 + \sqrt{w^2 + 4Q_W} \] ................................. V.

Equation V gives \( l \) in terms of the assigned or known data, \( w \) being among them.

Likewise from equation II we get finally (if \( l \) is known or assigned):
\[ w = -1/2 + \sqrt{1/4 + Q_l} \] ................................. VI

where
\[ Q_l = q(ni)^2/2swcT \tau \] ................................. VII.

Both equations III and VII are practically important although only auxiliary.

Expressions for \( Q_W \) and \( Q_l \) can be given other forms. Let us substitute for \( (ni) \) its value in equation I, then finally
\[ Q_W = qV^2/2swR_m(p+kw)cT \tau \] ................................. IIIa
\[ Q_l = qV^2/2swR_m(p+kw)cT \tau \] ................................. VIIa

where \( R_m \) stands for resistance of a mean turn at average temperature of the coil.

Likewise substituting from (4), it obtains that
\[ cT \tau = 1/A_1 \text{ and } A_1 = 1/cT \tau \] ................................. (14)

where \( A_1 \) stands for \( A/W \) and denotes area of the cooling surface per 1 watt dissipated; formulas III and VII, also IIIa and VIIa, transform to
\[ Q_W = qA_1(ni)^2/2sw = qA_1V^2/2swR_m(p+kw) \] ................................. IIIb
\[ Q_l = A_1(ni)^2/2sw = qA_1V^2/2swR_m(p+kw) \] ................................. VIIb.
Raising equation (6) to the rank of practical importance we write:

\[ n = \frac{\pi f}{q} \]

There are now three fundamental equations, namely, I, V (or VI), and VIII to determine four basic magnitudes pertaining to the coil, i.e., \( q \), \( \omega \), \( l \), and \( n \). This is practically advantageous because it permits a design to be adapted to other requirements arising from structural and economical considerations.

Suggestions and remarks. There are cases in which \( \omega \) must or can be estimated on the basis of some conditions or considerations; for example, the field coils of the winding reasonably assumed or estimated, equation I makes it possible to compute \( q \) for an assigned number of ampere-turns; then if necessary it is rounded to a wire-table figure. A more accurate value for \( \omega \) should then be computed from equation I substituting there the above table value. Now by using equations III (or IIIb) and V, \( l \) becomes computable.

Denoting ambient temperature by \( T \), outside and inside temperature rises by \( T_r \) and \( T_i \) respectively, and hottest spot and average temperature by \( T_h \) and \( T_a \) respectively, it obtains that

\[ T_h = T + T_r + T_i \]  \hspace{1cm} (15)

and also with sufficient accuracy that

\[ T_a = T + T_r + T_i/2 \]  \hspace{1cm} (16).

\( T_i \) usually varies from 5° to 30° C as ampere-turns change within the range 500 to 18,000. For a continual load in the absence of exceptional conditions it is suggested that \( 95° \text{C} \leq T_h < 100° \text{C} \). It is understood that if a load is intermittent or space limitations are stringent higher values of \( T_h \) are allowable, but in the latter case this would of necessity shorten the life of the coil. The cooling coefficient \( c \) usually varies from 0.005 to 0.010 depending on the nature of the ambient medium, conditions of ventilation, etc. Substituting values for \( T \), \( T_h \), and \( T_i \) in equations (15) and (16) one finds \( T_r \) and \( T_a \) which in combination with chosen \( c \) and \( \omega \) will set the system of fundamental equations and their auxiliaries to work. \( T_a \) can usually be chosen beforehand within the range 85° to 95° C if there are no exceptional conditions such, for instance, as drastic limitations of winding space.

Another approach to the development of a design is to begin with a reasonable assumption of \( A_i \) instead of the temperature and cooling coefficient; then equations IIIb and VIIb should be used. Usually \( A_i \) varies within the range 2 to 3.3, being about 2.65 for average cases. Also there are situations which require that \( l \) should be estimated first and then \( \omega \) determined from equations VI and VII. Since \( q \) is not yet known in this case, the space factor \( s \) should be estimated and checked afterwards.

The half-empirical formula

\[ T_i = 5 \sqrt{(n t)/600} \]  \hspace{1cm} (17)

for an estimation of \( T_i \) in degrees C at the beginning of work on a design is offered as helpful and adequately accurate.